

# Space-time Support System using Simplified Time-Change Fuzzy Set

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**Abstract**—In this paper, space-time fuzzy inference using “Simplified time-change fuzzy set” (sTC fuzzy set) is proposed aiming to simply process the ambiguity knowledge of time change, such as “Distance is gradually near” in a computer. The proposed sTC fuzzy set has four auxiliary sets, which enables the computer regarding the time change correctly in real time based on the expert knowledge. It is applied to the lane change support system to do an actual experiment. In the actual experiment, the computer determines the proper operation as a human supporter under a dynamic environment by using the proposed method. The obtained experiment results indicated that the proposed method can let the computer regard the time change correctly in real time. During the experiment the driver took appropriate actions to avoid potential danger based on the support information received.

## I. INTRODUCTION

The time change must be considered when supporting the actions of human life under a dynamic environment. For example, when the computer as a support person supports the lane change behavior in the city on the road which is an example of dynamic environment, how the states of vehicles around change over time must be grasped accurately in the computer.

A support system based on the knowledge of experts has been proposed so far, which using the time-change fuzzy set (TC fuzzy set) [5] to incorporate human knowledge about the time change into the computer. In reality, however, this method has some problems such as it is difficult to describe human knowledge in the computer and be applied in real time.

For the use in practice the improvement of the TC fuzzy set has been conducted so far [6]. In this research, we propose a simple time-change fuzzy set taking the TC fuzzy set as reference. This method is applied to the lane change support system, to verify that the appropriate support information can be sent to the driver in a real-time dynamic environment. By using the simplified time-change fuzzy set which is proposed in the paper, not only the current state, but also the ambiguity about the changes in the states up to the future, such as “Distance is gradually near”, can be simply incorporated into the computer.

## II. FUZZY INFERENCE CONSIDERING THE TIME-VARYING

### A. Fuzzy sets

Fuzzy is a technique proposed by L.A.Zadeh [1] for making computer dealing with the ambiguity which human is good at.

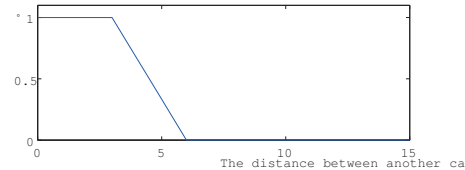


Fig. 1. An example of fuzzy sets(the distance between another car is near)

By using fuzzy sets, how many degrees a certain fact belongs to a set can be represented. The definition formula of fuzzy sets  $\tilde{X}$  is shown in equation (1)

$$\tilde{X}(x) = \int \mu_{\tilde{X}}(x)/(x), x \in R \quad (1)$$

An example of fuzzy sets defined by Zadeh is shown in Fig.1, which represents the feeling of mankind that “the distance between the vehicle is near”.

In order to give appropriate support information, it is important to let the computer that plays the role of a support person to think as an expert. Since fuzzy sets can be used to quantify the human knowledge and thinking, by using fuzzy sets, incorporating the knowledge of an expert into the computer is possible. For example, the fuzzy sets and control methods are applied to numerous intelligent cooperative control systems based on “fuzzy instruction” [4] and a “fuzzy target” [3].

### B. Time-change fuzzy set

Until now, the knowledge of an expert such as “the distance will be near soon” has been described in the computer by using the TC fuzzy set. The TC fuzzy set has an extension of the time domain by adding a time axis to the fuzzy sets defined by Zadeh as shown in Fig.1, in order to quantify the subjective feelings of mankind concerning the time change in the computer.

The definition formula of the TC fuzzy set  $\tilde{X}_{fn}$  regarding the state  $x(t)$  of the target is shown in equation (2).  $R$  is the whole sets of state quantities.  $\mu_{\tilde{X}_{fn}}(x, p)$  is the membership value regarding the state quantity  $x$  and the future time  $p$ .  $P$  is the maximum expected time.

$$\tilde{X}_{fn}(x, p) = \int_{R \times (0, P)} \mu_{\tilde{X}_{fn}}(x, p)/(x, p), \quad (2)$$

$$x \in R, p \in (0, P).$$

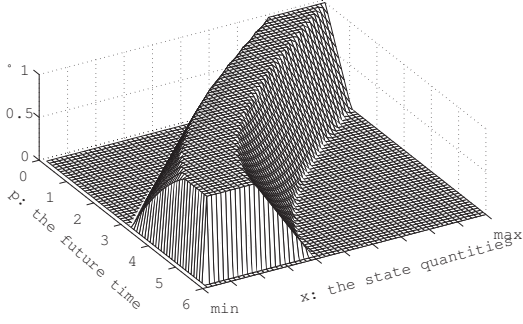


Fig. 2. An example of the time-change fuzzy set

The TC fuzzy set as shown in Fig.2 is used in order to incorporate the feeling of mankind that “the distance between another car is changing from far to near over time” to the computer. This is an example of the TC fuzzy set shown in equation (2). Specifically  $\mu_{\widetilde{X}'_{fn}}(x, p)$  in equation (2) is shown in equation (3).  $x$  is the state quantities of the target,  $p$  is the future time. So the membership value  $\mu_{\widetilde{X}'_{fn}}(x, p)$  is calculated based on the state quantities  $x$  and the expected time  $p$  by using the equation (3).

$$\mu_{\widetilde{X}'_{fn}}(x, p) = \min(1, \max(0, \min\{\frac{x}{3} + 1 - 4 \times (1 - \sin \frac{p\pi}{12}), -\frac{x}{3} + 2 + 4 \times (1 - \sin \frac{p\pi}{12})\})) \quad (3)$$

Fig.3 shows the decision-making process of the fuzzy control command according to the time-change fuzzy inference. Firstly the membership value regarding the current state is calculated based on the value of the external state. Secondly the value of the future state is predicted by the physical knowledge according to the value of the current state and the membership value regarding the future state is calculated based on the predicted value of the future state. The membership value regarding the transition of state is calculated based on the value of the current and future state at the same time. Finally each evaluation value is obtained to generate an appropriate support instruction.

### C. Simplified time-change fuzzy set

In this paper, the simplified time-change fuzzy set (sTC fuzzy set) is proposed to simplify the conventional TC fuzzy set. The sTC fuzzy set is constituted by four auxiliary sets to deal with the feeling of mankind about the time change in the computer. The auxiliary sets are constituted by the state of 0 seconds, the state of P seconds and the state transition. If  $R$  is the whole sets of state quantities as well, the simple time fuzzy sets  $\widetilde{X}'_{fn}$  regarding the state  $x(t)$  of the target can be

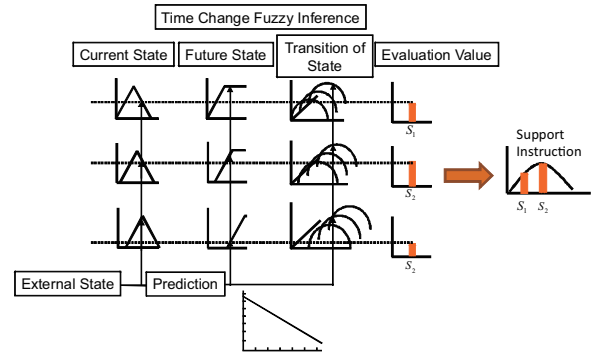


Fig. 3. The time-change fuzzy inference

defined as in equation (4).

$$\widetilde{X}'_{fn}(x, p) = \int_{R \times (0, P)} \{[\mu_{x_0}(x) \wedge \mu_{p_0}(p)] \oplus [\mu_{x_P}(x) \wedge \mu_{p_P}(p)]\} / (x, p), \quad x \in R, p \in (0, P). \quad (4)$$

$\mu_{x_0}$  here is the membership value of the state quantities  $x$  regarding the membership function of 0 seconds,  $\mu_{x_P}$  is the membership value of the state quantities  $x$  regarding the membership function of P seconds. Then  $\mu_{p_0}$  is the membership value of the expected time  $p$  regarding the state of 0 seconds in the state transition,  $\mu_{p_P}$  is the membership value of the expected time  $p$  regarding the state of P seconds in the state transition.

As an example of the sTC fuzzy set, four auxiliary sets of the sTC fuzzy set as shown in Fig.4 are used to deal with the language that “the distance between another car is changing from far to middle suddenly” in the computer. By the way the maximum expected time is 6 in this example.

The auxiliary set which represent the language that “the distance of 0 seconds is far” is defined by using the membership function of Fig.4(a). This membership function can be represented by using  $\pi$  type function as in equation (5). About the description of the membership function of the fuzzy sets, Yasunobu [2] defined the function descending from upper right to lower left as the S type function, because it is similar to the stretched shape from side to side of letter S. The function descending from upper left to lower right was defined as the Z type function. Furthermore the convex function overlapped with these was defined as the  $\pi$  type function. As for the parameters in each function,  $as$  and  $az$  represent the maximum satisfaction (The degree of membership function is 1) space.  $bs$  and  $bz$  represent the length of the space where the membership value is reduced to 0.5 from 1.0.

$$\mu_{x_0}(x) = \pi(x, as, az, bs, bz) = \pi(x, 11, 14, 0.5, 0.5) \quad (5)$$

“as, az, bs, bz” is the four parameters which are used to generate the membership function.

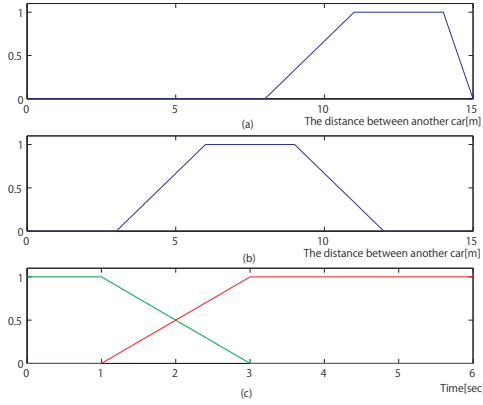


Fig. 4. The example of the auxiliary sets of the simple time-change fuzzy set

By using the membership function of the maximum expected time  $P$  in Fig.4(b), the auxiliary set which represent the language that “the distance of 6 seconds is middle” is defined. This membership function can be represented by the equation (6).

$$\mu_{x_6}(x) = \pi(x, as, az, bs, bz) = \pi(x, 6, 9, 1.5, 1.5) \quad (6)$$

$\pi$  type membership function is used in equation (6) as well. “as, az, bs, bz” are the four parameters.

The membership function of Fig.4(c) represents “the state transition is changing from the state of 0 seconds to the state of 6 seconds suddenly”. This membership function can be represented as equation (7).

$$\mu_{p_0}(p) = Z(p, az, bz) = Z(p, 1, 1) \quad (7)$$

$$\mu_{p_6}(p) = S(p, as, bs) = S(p, 3, 1) \quad (8)$$

$Z$  type membership function is used in equation (7) to define the auxiliary set which represents the state of 0 seconds. “az, bz” is the two parameters. And in equation (8)  $S$  type membership function is used to define the auxiliary set of 6 seconds. “as, bs” is the two parameters. Therefore, it is possible to define a sTC fuzzy set as shown in Fig.4 in the computer with only 12 parameters in total.

The membership function of the sTC fuzzy set is generated as shown in Fig.5 based on the equation (4) by using the above-mentioned four auxiliary sets.

#### D. Degree for simple time-change fuzzy set

As for how to determine the degree of the input that is given in the form of a function of time for the sTC fuzzy set, firstly the degree of input for the simple time fuzzy sets is calculated at each sampling time. Secondly the previous process is repeated until the maximum expected time. At last we make the arithmetic mean of all calculated degree as the final degree.

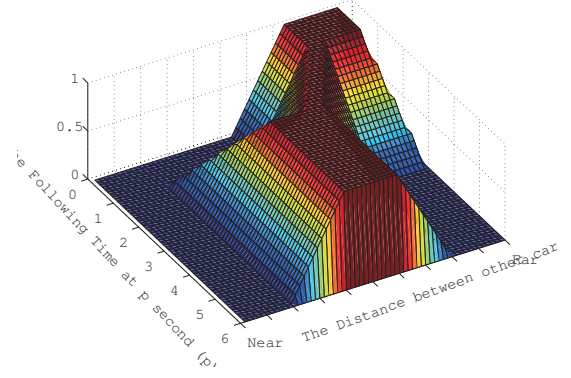


Fig. 5. The example of the membership function of the simple time-change fuzzy sets

A concrete example is given as following. The input which represents the trend that the distance between another car is changing from far to middle suddenly is shown in Fig.6. The horizontal axis is the future time  $p$ , and the vertical axis is the distance to another car  $x$ . The maximum expected time  $P$  is 6 seconds in this example. We can know that the distance to another car is changing suddenly from 15 meters to 5 meters from this graph. For comparison, four sTC fuzzy sets which represent the trend of distance as discussed in the previous section have been created with parameters of further four sets. The abbreviation, meaning and parameter values of each sTC fuzzy set are

FSM: The distance between another car is changing from Far to Middle Suddenly.  $\pi(x, 11, 15, 1, 1)$   
 $\pi(x, 5, 9, 1, 1)$   $Z(p, 1, 1)$   $S(p, 3, 1)$

FGM: The distance between another car is changing from Far to Middle Gradually.  $\pi(x, 11, 15, 1, 1)$   
 $\pi(x, 5, 9, 1, 1)$   $Z(p, 3, 1)$   $S(p, 5, 1)$

MSF: The distance between another car is changing from Middle to Far Suddenly.  $\pi(x, 5, 9, 1, 1)$   
 $\pi(x, 11, 15, 1, 1)$   $Z(p, 1, 1)$   $S(p, 3, 1)$

MGF: The distance between another car is changing from Middle to far Gradually.  $\pi(x, 5, 9, 1, 1)$   
 $\pi(x, 11, 15, 1, 1)$   $Z(p, 3, 1)$   $S(p, 5, 1)$

The degrees of the above-mentioned input for the four sTC fuzzy sets are calculated and the result of simulation is shown in Fig.6(b). Fig.6(b) shows that the membership value regarding the FSM sTC fuzzy set which means the distance is changing suddenly from far to middle is the highest one. And the membership value regarding the FGM sTC fuzzy set which means the distance is changing gradually from far to middle is a little lower than the FSM. It is the same as the above-mentioned human feeling that the membership value regarding the FSM sTC fuzzy set is the highest one in the four sTC fuzzy sets. So we can know that the feeling of mankind concerning the time change can be accurately grasped in the computer, by using the sTC fuzzy set proposed in this paper from the Fig.6.

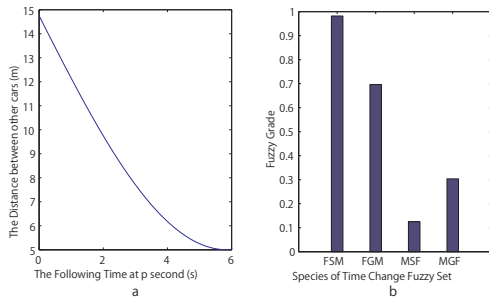


Fig. 6. The comparison of degrees

### III. APPLICATION TO THE LANE CHANGE SUPPORT SYSTEM

#### A. About the lane change

When you are driving a car on the road, the lane change is absolutely unavoidable, especially on the road with many lanes, where the relative position and velocity of other cars are changing every moment. Therefore in order to make a smooth lane change safely, the driving support system is required. When considering the human as a support person in such dynamic environment, the support person not only considers the current state but also predicts the future state, which is taken into account to determine the appropriate operating instruction.

#### B. Outline of the support

When performing a driving support the vehicle operation must be considered. In this paper, the vehicle operation about a lane change consists of “Driving in the current lane”, “Moving to the left lane”, “Moving to the right lane”. In order to give appropriate support information, the most important thing is the driving knowledge. Therefore it has to be examined that how a skilled driver thinks when making a lane change. When a skilled driver makes a lane change, firstly he will make sure that whether there is enough space to let the car move in. Secondly by understanding the relationship between his car and the car next to his car, he can finally make a smoothly lane change. In this paper, the driving knowledge is determined based on such skilled drivers. Considering the car in the lane where you want to move in, the fuzzy rules are determined as follows.

- Rule1: If the front car is far away and the behind car is far away, then moving to the right lane.
- Rule2: If the front car is far away and the behind car is changing from far to near suddenly, then driving in the current lane.
- Rule3: If the front car is changing from far to near gradually and the behind car is changing from far to near suddenly, then driving in the current lane.

#### C. System configuration

The configuration of this intellectual operation support system is shown in Fig.7. This support system consists of the

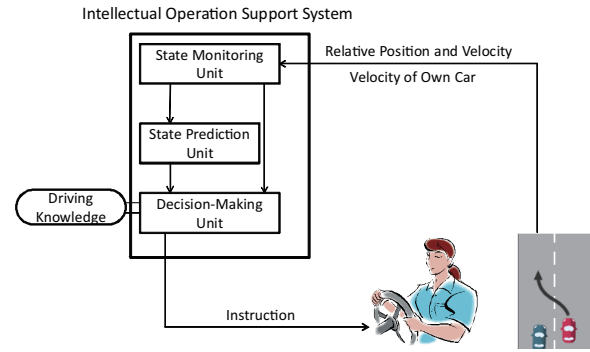


Fig. 7. The intellectual operation support system configuration

state monitoring unit, the state prediction unit and the decision-making unit. The relative position, relative velocity and the own vehicle speed are treated as the state quantities in the state monitoring unit. The prediction of the future state from the current state is performed based on the model of the laws of physics in the state prediction unit. In the decision-making unit, the propriety of the lane change considering the state of future is given to the driver based on the driving knowledge of the skilled drivers by using the sTC fuzzy set.

### IV. ACTUAL EXPERIMENT

#### A. Experimental environment and outline

The experiment is performed based on the proposed system by using the simple driving simulator(Fig.8). The support information of safety degree is obvious by using different colored characters. The case in which the vehicle in the left-hand lane on a two-lane roadway enters the right-hand lane is assumed. The initial position of the own car is 0[m]. The initial position and velocity of the other cars are appropriately given so that the effectiveness in real time can be verified.

#### B. Evaluation of effectiveness of the supporting information

Under different circumstances, the validity of the support information which is presented to the driver based on the proposed support method concerning the time change is verified through the following experiment.

- The own car : CASE1: The initial velocity 45(km/h)  
CASE2: The initial velocity 90(km/h)
- The front car : The initial position 100(m)  
The initial velocity 40(km/h)
- The behind car : The initial position -1000(m)  
The initial velocity 45(km/h)

In order to investigate the relationship between the relative distance and the time when the support information is sent to the driver, it is taken into account that the relative distance between the own car and the front car. The relationship between the relative distance and the time when the support information is sent to the driver under different relative velocity is shown in Fig.(9,10). The horizontal axis is the relative distance to the front car. The vertical axis is the category of the safety degree support information.



Fig. 8. The experiment scenery

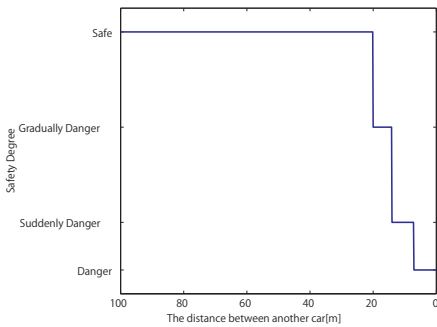


Fig. 9. The relationship between relative velocity and support information in low speed

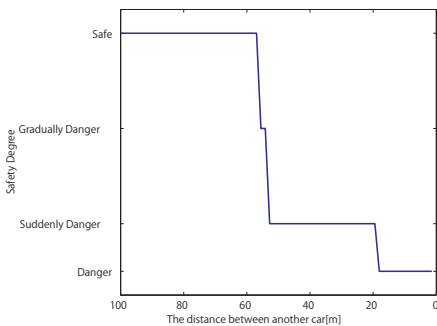


Fig. 10. The relationship between relative velocity and support information in high speed

From these graphs we can know that the faster the relative velocity is, the wider the danger zone will be. Particularly if the relative velocity was very fast like CASE2, the support

information “gradually danger” would be given in the distance. Besides it switched to “suddenly danger” immediately to give a warning to the driver after “gradually danger” was given. Under such a mechanism like this more appropriate and flexible support information can be given by changing the width of the danger zone and different region according to the different relative velocity. The driver can perform an action ahead of time even if the relative velocity is very fast by using a driving support system like this.

### C. Comparison with support system which takes into account only the current state

In order to verify the effectiveness of the proposed support method, we prepared a support system for comparison that gave support information by considering only the current state (hereafter referred to as “the conventional system”).

The conventional system only considers the relative distance to another car. If the relative distance is less than 20 meters, the support information will be “danger”. And if the relative distance is more than 20 meters, the support information will be “safe”. The operator will determine the operation and do it after comprehending the support information from the image.

In this experiment we assume a relatively dangerous situation, that the relative velocity between the own car and the front car is  $45(km/h)$ . The traffic situation is shown below. The tester who is a paper driver will make a lane change from the left lane to the right lane by using both of the conventional system and the proposed system under the following traffic situations. We evaluate whether the collision occurred during the lane change in this experiment.

The own car : The initial velocity  $85(km/h)$

The front car : The initial position  $100(m)$

The initial velocity  $40(km/h)$

The behind car : The initial position  $-1000(m)$

The initial velocity  $45(km/h)$

The result in the case of using the conventional system is shown in Fig.11. This is a representative example of the results. The horizontal axis is the relative distance to the front car. The vertical axis on the left shows the position of the lane. The vertical axis on the right shows the safety degree that is given by the support system. The solid line is the operation of the tester, the long dashed line is the position of the target lane and the short dashed line shows the support information. From this graph we can know that the driver started making a lane change when the relative distance to the front car was 30 meters. Because the relative velocity was fast, the relative distance fell within the range of 20 meters immediately, and the support information changed from “safe” to “danger” suddenly. From the graph we can know that because the driver was caught off guard, he could not handle the situation even if he had received a “danger” support information. Eventually the car crossed the center line and collided with the front car.

The result in the case of using the proposed support system is shown in Fig.12. The horizontal axis is the relative distance to the front car. The left vertical axis is the same as the

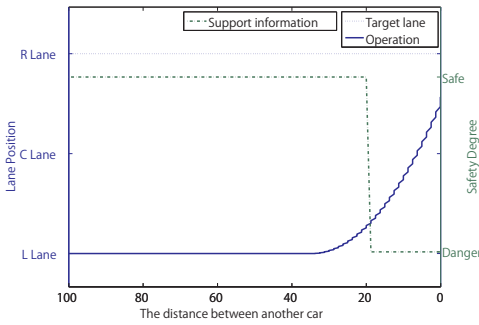


Fig. 11. Lane change behavior with the conventional system

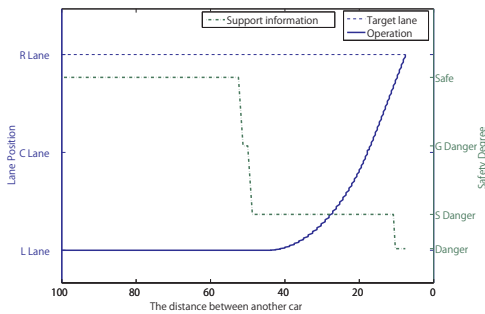


Fig. 12. Lane change behavior with the proposed system

previous example. The right vertical axis shows the safety degree that is given by the support system. The solid line is the operation of the tester, the long dashed line is the position of the target lane and the short dashed line shows the support information. From this graph we can know that the support information was switched from “safe” to “suddenly danger” when the relative distance to the front car was about 50 meters. The driver started making a lane change while decelerating referring to the support information that was presented, when the relative distance was about 40 meters. Finally the car completely moved to the right lane and completed the lane change safely when the relative distance was about 10 meters.

From the experimental results, we can know that the driver can find an appropriate path in advance before making a lane change by using the proposed support system, and it is possible to avoid collision safely. So it is verified that the proposed support method is more effective than the conventional one.

## V. CONCLUSION

In this paper, the simple time change fuzzy sets are proposed. By the simple time change fuzzy sets, the feeling of mankind concerning the time change such as “the distance will be near soon” can be dealt easily in the computer. This set is verified that it can be processed in real time under a dynamic environment. The proposed simple time change fuzzy sets are applied to the lane change support system to let computer determine the appropriate operation as a human expert. It is verified that the operator can make an appropriate

operation based on the support information that is given by the computer.

## ACKNOWLEDGMENT

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