A General Practical Design Method for Fuzzy PID Control from Conventional PID Control

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Abstract— This paper proposes a practical design method of fuzzy proportional-integral-derivative(PID) control system. Like conventional PID controller there are several kinds of fuzzy PID controllers. Being as simple structure, the research on how to choose the type of conventional PID controllers for different controlled plants is successful. Based on the analysis of relationship between conventional PID controller and fuzzy PID controller, we proposed a method on how to choose the type of fuzzy PID controller suitable for the controlled plant. Firstly the selected fuzzy PID controller is designed to have the same control property with corresponding PID controller because the parameters of its control rule base are calculated according to tuned parameters of PID controller. Then parameters of this fuzzy rule base of fuzzy PID controller are tuned by sequential quadratic programming (SQP) algorithm. The tuned control surface is nonlinear corresponding to the property of controlled plant. This method is applied to linear and nonlinear controlled plants. Simulation results show that it is an effective method to design fuzzy controller.

I. INTRODUCTION

It is well known that up until now, a conventional proportional integral-derivative (PID)-type controller is most widely used in industry due to its simple control structure, easy of design and inexpensive cost. According to a survey of the state of process control systems in 1989 conducted by the Japan Electric Measuring Instrument Manufacture's Association, more than 90% of the control loops were of the PID type[1]. The PID formulas are simple and can be easily adopted to corresponding to different controlled plant but it cannot yield a good control performance if controlled system is highly order and nonlinear.

Although many control applications show that a fuzzy logic controller can improve control performance especially for highorder linear or non-linear system, one of the fundamental problems in fuzzy control technology is that there lacks a design theory on fuzzy controllers. Many fuzzy controllers have been constructed, instead of systematically designed, case by case using the trial-and-error method guided by designers, experiences on fuzzy control. Some authors have reported fuzzy control relative to PID control [2]and some of them also do the stability analysis on designing the scaling factor of fuzzy PID controller[3], but their conclusions are individual and do not supply more on how to choose the type of the fuzzy PID control and how to design fuzzy rule base according to a tuned PID control.

In this paper we proposed a systemic methodology on how to design a fuzzy PID control in which both of the two aspect information sources are used effectively. Attention is focused on the following approaches:

 Based on the analysis of relationship between PID control and fuzzy control, we firstly proposed a method on how to choose the type of fuzzy PID controller suitable for the controlled plant.

- A method on how to design a two-term fuzzy control rule base with the control surface behaved the same linearity as the tuned two-term conventional control is depicted. The analysis is main on fuzzy PI controller because we have done the same analysis on fuzzy PD control in our previous work[6].
- Fuzzy rule base behaving linearity is tuned by Sequential Quadratic Programming method. The tuned control surface is nonlinearity corresponding to the property of controlled plant.
- The proposed method is used to linear and nonlinear controlled plant and simulation results show the practicality and reliability of this method.

II. FUZZY PID CONTROLLER AND ITS CONSTITUTE

It is well known that there are two types of parameters for fuzzy PID controllers. One is the scaling factors, the other is the fuzzy rule base. As being proven in our previous work[6], if we use the linear defuzzification algorithm with two inputs partitioned into uniformly distributed fuzzy regions, and if we choose suitable scaling factors and fuzzy rule base, a linear-like fuzzy control surface can be generated by linearly defined control rules in simplified inference method. It has been also proven that a fuzzy PI or PD controller is equivalent to a conventional PI or PD controller when suitable parameters are chosen. Because a detail analysis about fuzzy PD controller has been given in our previous work, we will focused on the fuzzy PI controller analysis as following.

A control surface with x, y and z (x example input $\triangle e$, y example input e and z example output $\triangle u$) expressed as:

$$k_p x + k_i y - z = 0 \tag{1}$$

A linear-like fuzzy PI control base with $(2n + 1) \times (2n + 1)$ partitions is defined as following:

- 1. The actual range of universe of discourse for each of two decision variable is $[-L_1, L_1], [-L_2, L_2]$ respectively;
- 2. For each decision variable the fuzzy sets is 2n+1 that is $(mf_{-n}, mf_{-(n-1)}, \dots, mf_{-1}, mf_0, mf_1, \dots, mf_{(n-1)}, mf_n)$;
- 3. The membership function of fuzzification is as Fig.1 (*L* example L_1 and L_2), the apex of each triangular membership function is $(-L, \frac{-(n-1)}{n}L, ..., \frac{-1}{n}L, 0, \frac{1}{n}L, ..., \frac{(n-1)}{n}L)$



Fig. 1. The fuzzification membership function of linear-like fuzzy logic controller.



Fig. 2. Linear-like fuzzy PI control surface.

L); and

4. The simplified inference method is used and the $(2n + 1)^2$ fuzzy rules are defined as following:

1.) if $\triangle e$ is mf_{-n} and e is mf_{-n} then $\triangle u$ is -R; 2.) if $\triangle e$ is $mf_{-(n-1)}$ and e is mf_{-n} then $\triangle u$ is $-R + \frac{R-S}{2n}$;; 2n.) if $\triangle e$ is $mf_{(n-1)}$ and e is mf_{-n} then $\triangle u$ is $-R + (2n-1)\frac{R-S}{2n}$; 2n+1.) if $\triangle e$ is mf_n and e is mf_{-n} then $\triangle u$ is -S; 2n+2.) if $\triangle e$ is mf_{-n} and e is $mf_{(n-1)}$ then $\triangle u$ is $-R + \frac{R+S}{2n}$;; 2(2n+1).) if $\triangle e$ is mf_n and e is $mf_{(n-1)}$ then $\triangle u$ is $S + \frac{R+S}{2n}$;

 $(2n+1)^2$.) if $\triangle e$ is mf_n and e is mf_n then $\triangle u$ is R. The control surface equation is:

$$\frac{R-S}{2L_1}x + \frac{R+S}{2L_2}y - z = 0$$
 (2)

if

$$R = L_1 k_i + L_2 k_p \tag{3}$$

$$S = L_2 k_p - L_1 k_i. (4)$$

If suitable L_1 and L_2 are selected, a fuzzy PI controller behaving linear-like fuzzy control surface as Fig.2 can be easily acquired which has the same control property as the tuned conventional PI controller. For example, when e and Δe are defined as 3×3 partition rules, a standard fuzzy PI rule base is as Table

TABLE I Linear fuzzy PI control rule base with 3 \times 3 partitions.

$\bigtriangleup u$	e		
$\triangle e$	-R	$-\frac{R-S}{2}$	S
	$-\frac{R+S}{2}$	0	$\frac{R+S}{2}$
	-S	$\frac{R-S}{2}$	R

TABLE II Linear fuzzy PD control rule base with 3×3 partitions.

u	e		
$\triangle e$	-R	$-\frac{R+S}{2}$	-S
	$-\frac{R-S}{2}$	0	$\frac{R-S}{2}$
	S	$\frac{R+S}{2}$	R

I. In our previous work, we also given when e and $\triangle e$ are defined as 3×3 partition rules, a standard fuzzy PD rule base is as Table II if

$$R = L_1 k_p + L_2 k_d \tag{5}$$

$$S = L_2 \, k_d - L_1 \, k_p. \tag{6}$$

III. DESIGN METHOD

To design a fuzzy PID controller we should firstly choose the type of fuzzy PID controller from fuzzy PI, PD, fuzzy PI+D or fuzzy PD+I controller. The type of the fuzzy control structure should be suitable to controlled plant. But the research on how to choose is few, we have not find the paper given any hints or systemic methods until now. On the other hand, it has been well known for a conventional control, the PD mode is used in the case of large error to speed up response, whereas the PI mode is applied for small error conditions to eliminate the steady-state offset[7]. It has also been proved that the property of of each type of Fuzzy PID control is corresponding to the type of conventional control respectively[8].

Many methods on how to tune a conventional controller have been proposed. Based on the method stated before, the tuned PID controller parameters can be used as the basis of selecting fuzzy PID controller using the relationship between a fuzzy PID controller and its corresponding PID controller. This method can be stated as follows, when a conventional PID are tuned, three parameters k_p , k_d and k_i are acquired. Observed these three parameters, if one of the parameter is much smaller than the other two parameters, for example if k_i is much smaller than k_p and k_d , a fuzzy PD controller should be selected priory. If there is no a clear difference, a three-term fuzzy control like fuzzy PI+D or fuzzy PD+I should be considers because it could get a better control result.

After the type of fuzzy PID controller is decided, the next should define the initial value of the parameters of the fuzzy PID controller which has also been known as one of the difficult problems in designing a fuzzy PID controller. Based on the analysis stated in Section II, conventional PID control behaves linear surface but fuzzy control surface can be either nonlinear or linear surface. In view of control surface conventional PID control can be thought as a special state of fuzzy control. That is why we proposed the method to design fuzzy control rule base from the tuned parameters of the conventional PID control.

Based on the method stated in Section II, a fuzzy PID controller which behaves the same linear control surface as the tuned conventional PID controller is easily tuned. Then the fuzzy rule base can be tuned to corresponding the controlled plant in a nonlinear surface. If the selected fuzzy controller are two-term fuzzy PD or fuzzy PI controller, only tune the fuzzy rule base, if the selected fuzzy controller is three-term fuzzy PD+I or fuzzy PI+D, keep k_i or k_d the same as conventional control and tune the fuzzy PD or fuzzy PI rule base.

In order to get the nonlinear control surface, the parameters of input membership functions and output membership functions of the fuzzy rule base should be tuned by a nonlinear optimization. In this research a sequential quadratic programming (SQP) algorithm as presented in MATLAB[5] is used which has been said that represent state-of-the-art in non-linear programming methods, because a non-linearly constrained problem can often be solved in fewer iterations using SQP than an unconstrained problem. One of the reasons is that due to the limits on the feasible area, the optimizer can make well informed decisions regarding directions of search and step length.

The parameters of optimization include the triangular membership functions in antecedent and singleton output. If the approximation quality is the only objective of optimization, the search space of the parameters to be optimized is not limited. Though the approximation quality for such an approach can expected to be quite good, the interoperability of the fuzzy rules may be get lost. This is due to the fact, that if the range of the membership functions is not restricted, often widely overlapping membership functions give good numerical results. Fuzzy membership functions lacking any physical interpretation and loosing locality are possible. To avoid this, different kinds of constraints should be put on the optimization: kinds of constraints, inequality constraints and parameter bounds[4]. SQP efficiently solve this constrained nonlinear optimization problem in which the objective function and constraints may be nonlinear functions of variables.

IV. SIMULATION

In the following simulation, all of the control rule base is in 3×3 partitions as in Table I and Table II as PI or PD controller respectively.

Example 1 (*A time-delay system*)

A first-order lag system with dead time is firstly investigated.



Fig. 3. PID and fuzzy control results on the first-order lag system.

The transfer function of the model is given by

$$G(s) = \frac{Ke^{-Ls}}{1+Ts} \tag{7}$$

where L is a length of dead time, T is a time constant of firstorder lag, and K is a plant gain. As the value of L/T increases, the system becomes dependent on dead time, the control of the system becomes difficult. In this simulation K=1, L=1, T=0.1that is L/T=0.1.

Using SQP tune PID parameters, we get k_p =0.39, k_d =0.04 and k_i =0.79 and the control result is in Fig.3. Observing these three parameters, k_d is much smaller than k_p and k_i . So simply use k_p and k_i to construct a conventional PI controller and its control result is also in Fig.3. A Linear fuzzy PI controller rule base is also designed using k_p and k_i , then this fuzzy rule base is tuned by SQP. Control results of these three controllers are shown in Fig.3. Fuzzy PI control is better than conventional PI and PID control. This example shows when k_d is much small, a fuzzy PI control performs a good performance.

Example 2 (A high-order system)

A third-order-linear model in laplace transfer function is described as

$$\frac{5}{s^3 + 4.5s^2 + 5.5s + 15}\tag{8}$$

This is a stable linear system. The tuned conventional PID controller parameters by SQP are k_p =4.82, k_d =3.5, k_i =3.82 and the tuned PD and PI controller parameters are k_p =15, k_d =7.5 and k_p =0.05, k_i =0.833 respectively. From the tuned PID parameters, it is difficult to decide which one we should choose from fuzzy PD control or fuzzy PI control because their parameters have not so much difference as the example 1. So we do the simulation in three controllers: a fuzzy PD controller, a fuzzy PI controller and a three-term fuzzy PD+I controller. These three kinds of fuzzy control results and responding conventional PID control results are shown in Fig.4,Fig.5 and Fig.6. From Fig.4, Fig.5 and Fig.6, PI fuzzy control behaves better control performance than fuzzy PD+I and fuzzy PD control. This example show, when there is no clear difference of the value k_p , k_d and k_i , it is very difficult to decide which one we should choose



Fig. 4. PID and fuzzy PD+I control results on the three-order system.



Fig. 5. PD and fuzzy PD control results on the three-order system.

from fuzzy PI controller or fuzzy PD controller. In this example, Fuzzy PI controller get the best result.

Example 3 (nonlinear system)

Consider the nonlinear process governed by

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} + 0.25y^2 = u(t - L)$$
(9)

where L=0.5. This plant is controlled by Conventional PID controller and after tuning the parameters of PID by SQP, we have $k_p=1.56$, $k_d=1.33$, $k_i=0.17$ and the control result is in Fig.7. Because k_i is much small than k_p and k_d , a conventional PD controller is chosen and tuned by SQP, we get $k_p=2.05$, $k_d=1.45$; the control result is also in Fig.7. A fuzzy PD controller is constituted based on this PD controller, then control rule base is tuned by SQP. Control result is also in Fig.7.

V. CONCLUSION

In this research we proposed a pratical design method for fuzzy PID control and three designing examples are given both from linear and nonlinear plants. Simulation results show it is a effective method, the advantage of this method is that it both use the information from data and expert knowledge. Tuning the parameters of the fuzzy PID controller is not so tedious by the proposed method because much of work is done by computer. The proposed method is suitable for the situation which a system model is reliable. Meanwhile, it has been shown that the tuning



Fig. 6. PI and fuzzy PI control results on the third-order system.



Fig. 7. PID control, PD control and Fuzzy PD control results on the nonlinear system.

of a fuzzy PID controller is much more complex than a conventional PID controller. Fuzzy controller should seek applications where conventional control technologies perform poorly.

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